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LAMINAR MIXED CONVECTION IN AN ISOTHERMAL HORIZONTAL TUBE: CORRELATION OF HEAT TRANSFER DATA

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Abstract—A semi-analytical correlation for laminar mixed convection in an isothermal, horizontal tube is developed and found to describe all the available heat transfer data (excluding that of Kern and Othmer) with an RMS deviation of 11.7% based upon log-mean Nusselt number, 11.0% based on arithmetic-mean Nusselt number and 9.8% based on fractional bulk-temperature rise. The fact that the correlation does not describe the bulk of the Kern and Othmer data is attributed to the correspondingly large Rayleigh numbers, for which further investigations will be required. Although derived for a large-Prandtl-number fluid, the correlation is found to equally well describe available buoyancy-dominated heat transfer in air.

NOMENCLATURE

a,	tube radius;
C_1,	0.87052, equations (8a) and (8b);
C_p,	specific heat;
F_1,	correction factor, equation (18);
g,	gravity;
Gr,	Grashof number, $g\beta\|\Delta T\|a^3/\nu^2$;
h,	heat transfer coefficient;
k,	thermal conductivity;
L,	tube length;
L_{devel},	developing length (in experiments);
$\dot{m}$,	mass flow rate;
$\overline{Nu}$,	average Nusselt number, $\bar{h}a/k$, with $\bar{h}$ based on ΔT;
$\overline{Nu}_{am}$,	arithmetic-mean Nusselt number (based on $T_w - T_a$);
$\overline{Nu}_{lm}$,	log-mean Nusselt number (based on local $T_w - T_b$, averaged);
Pr,	Prandtl number, $\mu C_p/k$;
Re,	Reynolds number, Wa/ν;
T_a,	average of bulk temperatures at inlet and outlet;
T_b,	bulk temperature (either local or at $z = L$, depending on context);
T_0,	uniform inlet temperature;
T_w,	uniform wall temperature;
W,	average axial velocity;
z,	axial coordinate.

Greek symbols

β,	coefficient of volumetric thermal expansion;
ΔT,	$T_w - T_0$;
ΔT^*,	$T_w - T_b$;
ΔT_b,	$T_b - T_0$;
μ,	dynamic viscosity;
ν,	kinematic viscosity;
ξ,	$z/(a\,Re\,Pr)$;
ρ,	fluid density;
σ,	$(Gr\,Pr)^{1/4}\,\xi$;
ϕ,	$\Delta T_b/\Delta T$.

Subscripts

a,	evaluated at T_a;
B,	buoyancy-induced;
F,	forced-flow induced;
L,	evaluated at $z = L$;
w,	evaluated at wall.

1. INTRODUCTION

A SURVEY of the heat transfer literature indicates that the correlations usually recommended for laminar mixed convection in horizontal isothermal tubes are those of either Oliver [1], Brown and Thomas [2] or Depew and August [3], even though none of these empiricisms describes all the available data particularly well. On the other hand, it has been recently shown [4] that a composite result based upon the boundary-layer analysis [5] and the finite-difference results from ref. [6] can correlate the natural-convection-dominated data in ref. [1–3] quite well.

In the present paper, it is shown that the buoyancy-dominated, thermal-boundary-layer series expansion obtained in ref. [4] can be simply expressed in terms of a single, closed-form result. When this latter natural-convection-dominated asymptote is combined with the forced-flow-dominated (Leveque–Graetz) asymptote in the manner of Churchill [7], it is found that the resulting correlation for the log-mean Nusselt number can describe the data in refs. [1]–[3], together with those of refs. [8–11], with an overall RMS deviation of 15%. Further, by empirically adjusting the constants in the buoyancy-dominated asymptote, the above deviation can be reduced to 12%. This represents a substantial improvement over the correlations from [1–3] which describe the above-cited data with an RMS deviation of 25%, 24% and 24%, respectively.

On the other hand, it is found that the above proposed correlation does not describe most of the data of Kern and Othmer [12], with the poorest agreement ($\approx$35–50% RMS deviation) being with the data for which the Rayleigh number is greatest (as large as 2×10^8, based on tube radius, which is considerably larger than in the other investigations). The implication is that a different buoyancy-dominated mode was present in ref. [12], for which further independent experimental investigation would be desirable.

Finally, although the buoyancy-dominated theory developed in ref. [4, 5] is based upon a large Prandtl number fluid, it is found that the present correlation can equally well describe the buoyancy-dominated data in air reported by Jackson *et al.* [13, 14].

2. PREVIOUS CORRELATIONS

In considering the various heat transfer correlations for laminar mixed convection in a horizontal isothermal tube, mention should be made of the early study by Eubank and Proctor [15] which, based upon the then available data in oils, resulted in the following expression (in present notation):

$$\left(\frac{\mu_w}{\mu_a}\right)^{0.14}\overline{Nu}_{am} = 0.875\left\{\frac{\pi}{\xi_L} + 12.6 \times \left[16\left(\frac{2-\phi}{2}\right)Gr\,Pr(a/L)\right]^{0.40}\right\}^{1/3} \quad (1)$$

McAdams [16] subsequently modified this result as follows:

$$\left(\frac{\mu_w}{\mu_a}\right)^{0.14}\overline{Nu}_{am} = 0.875\left\{\frac{\pi}{\xi_L} + 0.04 \times \left[16\left(\frac{2-\phi}{2}\right)Gr\,Pr(a/L)\right]^{0.75}\right\}^{1/3} \quad (2)$$

Later correlations, proposed successively by Oliver [1], Brown and Thomas [2] and Depew and August [3], are given respectively as follows:

$$\left(\frac{\mu_w}{\mu_a}\right)^{0.14}\overline{Nu}_{am} = 0.875\left\{\frac{\pi}{\xi_L} + 5.6\times10^{-4} \times \left[4\left(\frac{2-\phi}{2}\right)Gr\,Pr(L/a)\right]^{0.70}\right\}^{1/3}, \quad (3)$$

$$\left(\frac{\mu_w}{\mu_a}\right)^{0.14}\overline{Nu}_{am} = 0.875\left\{\frac{\pi}{\xi_L} + 0.012 \times \left[2\pi\left(\frac{2-\phi}{2}\right)^{1/3}\frac{Gr^{1/3}}{\xi_L}\right]^{4/3}\right\}^{1/3}, \quad (4)$$

$$\left(\frac{\mu_w}{\mu_a}\right)^{0.14}\overline{Nu}_{am} = 0.875\left\{\frac{\pi}{\xi_L} + 0.12 \times \left[2\pi\left(\frac{2-\phi}{2}\right)^{1/3}\frac{Gr^{1/3}\,Pr^{0.36}}{\xi_L}\right]^{0.88}\right\}^{1/3}. \quad (5)$$

It is noted that the form of each of the above is modelled after the earlier semi-analytical result obtained by Martinelli and Boelter [17] (or see ref. [18]) for the vertical tube case. Coincidentally, this form is the same as that advocated by Churchill [7] for mixed convection in general, namely

$$\overline{Nu} = \{\overline{Nu}_F^3 + \overline{Nu}_B^3\}^{1/3}. \quad (6)$$

Lastly, it might be noted that the Heat Transfer Data Book [19], available from General Electric Company, recommends the following for the present application:

$$\left(\frac{\mu_w}{\mu_a}\right)^{0.14}\overline{Nu}_{lm} = C_{HORTZ} \times (\overline{Nu}_F)_{lm} \quad (7a)$$

where

$$(\overline{Nu}_F)_{lm} = 1.83 + \frac{0.114/\xi_L}{1 + 0.04(4/\xi_L)^{0.8}} \quad (7b)$$

and

$$C_{HORIZ} = \left\{1 + \left(\frac{0.0083}{\pi}\right)\left[8\left(\frac{2-\phi}{2}\right)Gr\,Pr\right]^{3/4}\xi_L\right\}^{1/3} \quad (7c)$$

3. NEW CORRELATIONS

As shown in ref. [4], the buoyancy-dominated bulk-temperature rise in horizontal, isothermal tubes can be described by the following composite expression, based upon the results in refs. [5] and [6]:

$$\left(\frac{\Delta T_b}{\Delta T}\right)_B \equiv \phi_B = \begin{cases} \sum_{n=1}^{6} C_n \sigma_L^n, & \sigma_L \leq 0.70 \\ \sum_{n=0}^{5} D_n \sigma_L^n, & 0.70 < \sigma_L \leq 6.0 \end{cases} \quad (8a)$$

where, for $n = 1, 2, \ldots, 6$

$$C_n = 0.87052,\ -0.47363,\ 0.20615,\ -0.07851,\ 0.02734,\ -0.00892 \quad (8b)$$

and, for $n = 0, 1, \ldots, 5$

$$D_n = 0.00369,\ 0.80669,\ -0.31435,\ 0.066911,\ -0.0073590,\ 0.00032559. \quad (8c)$$

In particular, the upper expansion in equation (8a) is based upon a buoyancy-dominated thermal boundary layer which interacts with a non-stratified core, as developed in ref. [5], whereas the lower series is based upon a least-square fit of the finite-difference results from ref. [6], as rescaled in ref. [4]. On the other hand, an accurate representation of the forced-flow-dominated bulk-temperature rise, based upon a composite 4-term Leveque–4-term Graetz expansion, is given by

$$\phi_F = \begin{cases} \sum_{n=1}^{4} a_n \xi_L^{(n+1)/3}, & \xi_L \leq \xi_1 \equiv 0.04 \\ \sum_{n=1}^{4} a_n \xi_1^{(n+1)/3} + \sum_{n=1}^{4} b_n (e^{-\alpha_n \xi_1} - e^{-\alpha_n \xi_L}), & \xi_L > \xi_1 \end{cases} \quad (9a)$$

where, for $n = 1, 2, 3, 4$:

$$a_n = 2.5638,\ -1.2000,\ -0.1767,\ -0.0889, \quad (9b)$$

and

$$\alpha_n = 3.65679,\ 22.3047,\ 56.9605,\ 107.620, \quad (9c)$$

$$b_n = 0.819050, 0.097526, 0.032504, 0.015440. \quad (9d)$$

Accordingly, a correlation based upon the above might be expressed in terms of $(\overline{Nu})_{\mathrm{lm}}$ as follows:

$$\left(\frac{\mu_w}{\mu_a}\right)^{0.14} (\overline{Nu})_{\mathrm{lm}} = \{(\overline{Nu}_F)^3_{\mathrm{lm}} + (\overline{Nu}_B)^3_{\mathrm{lm}}\}^{1/3} \quad (10)$$

where $(\overline{Nu}_F)_{\mathrm{lm}}$ and $(\overline{Nu}_B)_{\mathrm{lm}}$ are obtained from equations (9) and (8), respectively, by using the relationship

$$\overline{Nu}_{\mathrm{lm}} = -\frac{\ln(1-\phi)}{2\xi_L}. \quad (11)$$

As a simplification of equation (8), it might be noted that the upper expansion in (8a) can actually be expressed as a single closed-form term which is applicable for arbitrary σ_L. That is, as is shown in Appendix 1, the upper expansion in (8a) can be replaced by the single term

$$\phi_B = 1 - \left(1 + \frac{C_1}{4}\sigma_L\right)^{-4} \quad (12)$$

where $C_1 = 0.87052$, as in equation (8b). The resulting plot of ϕ vs σ_L based upon (12) is shown by the solid line in Fig. 1 whereas corresponding results based upon equation (8) are shown by the dashed line. Also presented in Fig. 3 are corresponding Nusselt number curves which have been generated from ϕ. In this regard, it is noted from equation (11) that

$$\frac{\overline{Nu}_{\mathrm{lm}}}{(Gr\,Pr)^{1/4}} = -\frac{\ln(1-\phi)}{2\sigma_L} \quad (13)$$

whereas it can be shown that

$$\frac{\overline{Nu}_{\mathrm{am}}}{(Gr\,Pr)^{1/4}} = \left(\frac{2}{2-\phi}\right)\frac{\phi}{2\sigma_L} \quad (14)$$

and

$$\frac{\overline{Nu}}{(Gr\,Pr)^{1/4}} = \frac{\phi}{2\sigma_L}. \quad (15)$$

In particular, then, from equations (12) and (13) it follows that

$$\frac{(\overline{Nu}_B)_{\mathrm{lm}}}{(Gr\,Pr)^{1/4}} = \left(\frac{2}{\sigma_L}\right)\ln\left(1 + \frac{C_1}{4}\sigma_L\right). \quad (16)$$

Finally, in order to possibly improve agreement with the available data, a generalization of the theoretically based result, equation (16), will also be considered below, namely

$$\frac{(\overline{Nu}_B)_{\mathrm{lm}}}{(Gr\,Pr)^{1/4}} = \frac{2\ln\left(1 + \frac{n}{4}C\sigma_L\right)}{n\sigma_L} \quad (17a)$$

where n and C will be treated as adjustable parameters [note: this form is obtained by replacing C_1 with C in equation (A.2) and $(1-\phi)^{-5/4}$ by $(1-\phi)^{-(4+n)/4}$]. In particular, based upon comparison with the data in ref. [1–3] and [8–11], the following values have been chosen:

$$C = 0.87, \quad n = 2.2. \quad (17b)$$

4. COMPARISON WITH AVAILABLE DATA

Examination of the literature indicates that the most pertinent data is that reported in refs. [1–3] and [8–12]. The operating ranges of these experimental investigations, listed in chronological order, are given in Table 1 with Appendix 2 indicating the source of the property values used for the various fluids. Table 2 lists the percentage RMS deviation of these experimental data with the various correlations in Section 2, as given by equations (1)–(5) and (7), and with the three correlations of Section 3, where each of the latter is based upon equation (10) with $(\overline{Nu}_F)_{\mathrm{lm}}$ obtained from equations (9) and (11) and $(\overline{Nu}_B)_{\mathrm{lm}}$ based upon either equations (8) and (11) or equation (16) or (17), respectively.

The second set of values (in parentheses) shown in

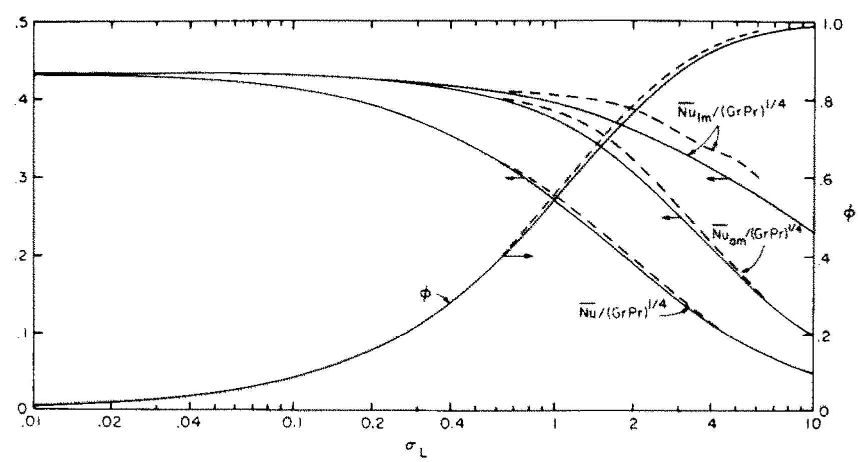

FIG. 1. Plots of ϕ, $Nu_{\mathrm{lm}}/(Gr\,Pr)^{1/4}$, $Nu_{\mathrm{am}}/(Gr\,Pr)^{1/4}$ and $Nu/(Gr\,Pr)^{1/4}$ vs σ_L where the solid curve for ϕ is based on equation (12) and the dashed curve on equation (8); the various Nusselt numbers are derived from ϕ based on equations (13), (14) and (15).

Table 1. Operating ranges of pertinent experimental investigations. All property values in Pr, Gr and Re have been evaluated at T_a

Investigator(s)	Symbol	# Data Points	Fluid	a(cm)	L/a	L_{devel}/a	heated/ cooled	W(cm/sec)	ΔT(°C)	Pr	$(Gr\ Pr)^{1/4}$	Re	$(\mu_w/\mu_a)^{.14}$
Holden [8]	◆	19*	light oil	0.627	239	99	heated	0.7, 51	53, 76	144, 379	15, 20	3.8, 163	0.76, .0.87
White [8]	◇	25	" "	"	223	219	"	92, 380	64, 78	364, 722	13, 15	135, 658	0.69, 0.76
R & H [9]	▽	29	94.5% glycerol/ water	0.357	439	174	"	2.4, 25	27, 68	141, 618	7, 9	3.8, 58	0.75, 0.92
S, K & M [10]	⬘	25	light oil	0.753	244	0	"	55, 180	46, 54	176, 216	20, 20	268, 1085	0.82, 0.84
" " "	⬗	28	" "	"	365	"	"	28, 181	46, 51	179, 194	20, 20	164, 1020	0.83, 0.84
" " "	⬖	26	" "	"	470	"	"	22, 168	43, 54	159, 200	20, 21	138, 1093	0.83, 0.86
S & T [11]	△	16	oil A	0.787	198	78	cooled	16, 138	-69, -33	152, 602	14, 25	32, 1050	1.29, 1.52
"	▲	17	oil B	"	"	"	heated	17, 234	38, 46	366, 573	15, 16	30, 618	0.79, 0.84
"	▲	25	oil C	"	"	"	cooled	25, 250	-100, -32	488, 14600	6, 21	2.0, 537	1.46, 2.13
K & O [12]	⦶	30	transformer oil	0.790	386	116	heated	15, 147	193, 227	51, 170	31, 42	345, 1506	0.62, 0.76
"	⦶	32	" "	1.529	200	60	"	6.6, 45	173, 218	53, 139	52, 67	256, 1113	0.65, 0.76
"	⦶	38	" "	3.137	97	29	"	0.8, 21	119, 190	38, 99	93, 121	97, 1624	0.70, 0.83
"	⊖	20	core oil	0.790	386	116	"	13, 86	195, 215	569, 4414	13, 24	19, 45	0.50, 0.68
"	⊖	30	" "	1.529	200	60	"	6.1, 38	152, 214	777, 2361	26, 35	15, 52	0.55, 0.65
"	⊖	34	" "	3.137	97	29	"	1.1, 11	141, 208	539, 1591	47, 63	9.8, 40	0.57, 0.73
"	○	15	cylinder oil	0.790	386	116	"	17, 100	176, 196	158, 395	25, 32	66, 427	0.66, 0.74
"	◒	30	" "	1.529	200	60	"	3.1, 34	148, 201	143, 458	38, 51	46, 173	0.64, 0.79
"	◓	30	" "	3.137	97	29	"	1.8, 12	129, 186	227, 395	68, 79	36, 241	0.66, 0.75
Oliver [1]	▼	15	glycerol	0.635	144	72	"	1.2, 7.1	8.8, 19	8590, 13000	3, 4	0.10, 0.35	0.84, 0.92
"	▼	4	" "	"	"	"	cooled	1.7, 3.7	-13, -12	16400, 17600	3, 3	0.06, 0.14	1,14, 1.15
"	▶	14	ethyl alcohol	"	"	"	heated	1.2, 12	12, 21	16, 17	22, 26	54, 524	0.96, 0.98
"	▶	7	" "	"	"	"	cooled	1.9, 12	-12, -10	17, 18	21, 22	72, 510	1.02, 1.03
"	◪	16	water	"	"	"	heated	0.7, 12	11, 21	5.5, 6.9	14, 19	57, 759	0.96, 0.98
"	◩	8	"	"	"	"	cooled	0.8, 12	-10, -9	7.9, 8.8	12, 12	42, 666	1.02, 1.03
"	▼	18	77% glycerol/ water	"	"	"	heated	1.4, 12	9, 21	258, 346	8, 10	3.2, 25	0.88, 0.96
"	▼	7	" "	"	"	"	cooled	1.5, 12	-12, -10	369, 461	7, 8	2.3, 19	1.08, 1.10
B & T [2]	◨	10	water	"	"	0	"	3.0, 4.5	-48, -12	3.6, 5.6	16, 28	229, 510	1.03, 1.09
"	◧	27	"	1.270	72	"	"	0.8, 3.1	-46, -15	4.1, 7.4	25, 44	118, 552	1.04, 1.11
"	⬓	38	"	0.635	216	"	"	3.0, 6.4	-56, -8	3.8, 6.8	13, 28	202, 626	1.02, 1.11
"	⬒	10	"	1.270	72	60	"	1.6, 2.3	-33, -13	4.3, 5.9	27, 39	242, 442	1.03, 1.07
D & A [3]	□	16	"	1.000	57	192	"	0.7, 8.7	-22, -10	5.7, 8.0	18, 26	58, 820	1.03, 1.07
"	▷	13	94% ethyl alcohol/water	"	"	"	"	0.6, 12	-21, -11	17, 21	28, 36	37, 780	1.03, 1.07

*Omitting run 55, for which ζ_L is unusually large and corresponding value of $(1 - \phi)$ is unreasonably high.

Table 2. Comparison of experimental data with various correlations

Investigator(s)	Symbol	Eqn (1)	Eqn (2)	Eqn (3)	Eqn (4)	Eqn (5)	Eqn (7)	Eqns (8,9,10,11)	Eqns (16,9,10,11)	Eqns (17,9,10,11)
Holden [8]	◆	25 (20)	50 (67)	27 (29)	31 (52)	20 (27)	71 (31)	23* (23*)	16 (16)	15 (14)
White [8]	◇	6 (6)	3 (3)	4 (4)	36 (36)	33 (33)	119 (10)	5 (6)	5 (6)	5 (6)
R & H [9]	▽	28 (17)	21 (41)	15 (10)	16 (36)	16 (13)	27 (16)	9 (9)	8 (8)	9 (8)
S, K & M [10]	⬘	17 (17)	25 (25)	20 (20)	38 (38)	24 (24)	155 (12)	14 (12)	14 (12)	14 (12)
" " "	⬗	11 (12)	26 (27)	12 (12)	34 (34)	25 (25)	113 (10)	8 (8)	8 (8)	7 (7)
" " "	⬖	21 (21)	45 (46)	18 (18)	25 (25)	17 (17)	119 (21)	8 (7)	8 (7)	9 (7)
S & T [11]	△	16 (16)	29 (29)	21 (21)	34 (34)	26 (26)	133 (15)	10 (9)	10 (9)	10 (9)
"	▲	26 (26)	35 (35)	31 (31)	21 (21)	17 (17)	173 (24)	28 (25)	28 (25)	28 (25)
"	▲	15 (15)	19 (19)	18 (18)	23 (23)	24 (24)	142 (14)	17 (15)	17 (15)	17 (15)
K & O [12]	ȸ	9 (9)	41 (42)	15 (14)	45 (45)	30 (30)	57 (14)	33 (34)	33 (33)	31 (31)
"	ȸ	21 (21)	49 (50)	13 (13)	44 (43)	18 (18)	67 (18)	38 (38)	38 (38)	35 (35)
"	ȸ	16 (16)	15 (15)	11 (10)	55 (55)	23 (25)	55 (32)	49 (49)	49 (49)	46 (46)
"	⊖	29 (31)	65 (67)	30 (32)	30 (32)	22 (22)	111 (37)	14 (14)	14 (14)	16 (16)
"	⊖	17 (17)	46 (47)	22 (22)	21 (21)	26 (26)	101 (15)	17 (17)	17 (17)	16 (16)
"	⊖	13 (14)	35 (37)	23 (24)	26 (26)	23 (24)	80 (12)	32 (32)	32 (32)	30 (30)
"	○	10 (10)	33 (34)	10 (10)	33 (33)	30 (30)	74 (9)	24 (24)	24 (24)	22 (23)
"	◒	23 (23)	21 (22)	25 (25)	46 (45)	40 (40)	24 (32)	49 (49)	49 (49)	47 (47)
"	◓	15 (15)	19 (19)	15 (15)	47 (47)	26 (26)	54 (31)	48 (48)	48 (48)	46 (46)
Oliver [1]	▼	33 (31)	16 (15)	17 (15)	16 (15)	30 (29)	7 (14)	16 (17)	16 (17)	16 (16)
"	▽	14 (13)	11 (13)	11 (12)	11 (13)	10 (9)	29 (15)	13 (13)	13 (13)	13 (13)
"	▶	20 (15)	45 (52)	18 (19)	19 (20)	10 (15)	40 (13)	25 (25)	23 (23)	14 (14)
"	▶	13 (10)	55 (64)	24 (29)	23 (27)	16 (22)	49 (21)	18 (18)	17 (17)	13 (13)
"	◪	40 (31)	33 (43)	24 (19)	21 (19)	17 (15)	31 (22)	31† (31†)	23 (23)	12 (12)
"	◩	36 (31)	22 (29)	14 (12)	22 (22)	18 (14)	28 (14)	21 (21)	19 (19)	12 (12)
"	▼	32 (30)	14 (17)	11 (12)	17 (19)	28 (27)	20 (11)	17 (18)	17 (17)	15 (15)
"	▼	33 (31)	8 (11)	6 (6)	9 (11)	24 (22)	20 (5)	15 (15)	14 (15)	11 (11)
B & T [2]	◨	22 (14)	51 (72)	10 (26)	7 (15)	22 (41)	25 (13)	22 (22)	17 (17)	10 (10)
"	◧	18 (16)	51 (62)	46 (56)	7 (7)	40 (52)	48 (22)	22 (22)	17 (17)	6 (6)
"	⬓	22 (13)	57 (81)	8 (16)	5 (18)	19 (38)	24 (13)	20 (20)	15 (15)	8 (8)
"	⬒	7 (9)	57 (67)	51 (60)	7 (3)	39 (48)	53 (25)	17 (17)	13 (13)	3 (3)
O & A [3]	□	21 (18)	39 (44)	42 (48)	31 (32)	19 (25)	59 (21)	17 (17)	15 (15)	11 (12)
"	▷	21 (22)	56 (59)	63 (66)	34 (35)	27 (30)	98 (29)	18 (18)	17 (17)	14 (14)
All data		21 (19)	38 (44)	23 (25)	32 (34)	25 (28)	84 (21)	27 (27)	26 (26)	24 (24)
All data but K & O [12]		23 (19)	38 (47)	25 (28)	24 (28)	24 (29)	92 (18)	18 (17)	15 (15)	12 (12)

*omitting one data point for which (8) is inapplicable due to $\eta_L > 6$

†omitting two data points for which (8) is inapplicable due to $\eta_L > 6$

Table 2 corresponds to modifications of the various correlations as follows. In the case of equations (1)–(5), the RHS has been multiplied by F_1, a correction factor developed in [17] (or see ref. [18]) which purports to correct for basing the Nusselt number upon the arithmetic-mean, rather than the logarithmic-mean, temperature difference and can be expressed as [1, 16–18]

$$F_1 = \frac{\phi \Big/ \left(1 - \frac{1}{2}\phi\right)}{\ln\left(\frac{1}{1-\phi}\right)}. \quad (18)$$

On the other hand, for the case of the correlation based on equation (7), the bracketed values in Table 2 correspond to replacing equation (7b) with

$$(\overline{Nu_F})_{lm} = 1.83 + \frac{0.1336/\xi_L}{1 + 0.04\,(4/\xi_L)^{2/3}} \quad (19)$$

which is an empirical fit developed by Hausen [20] (or see ref. [21]) for the Graetz solution. In this regard, it is noted that the General Electric Heat Transfer Data Book [19] bases the form of equation (7b) upon equation (15) of Kays [21], a result which corresponds to the case of a uniform inlet velocity, but then adjusts the coefficients in order to approximate the case of a fully-developed inlet velocity. Such a development seems to have been rather unnecessary, however, given that Kays [21] himself uses equation (19) for the latter case.

Finally, the bracketed values for the last three correlations in Table 2 are based upon replacing equation (9) by a simpler representation but one which is more accurate than equation (19). In particular, following the lead of Worsoe-Schmidt and Leppert [22], who considered a similar form for the local Nusselt number, we assume that

$$(\overline{Nu_F})_{lm} = 1.282\,\xi_L^{-1/3}\,e^{-\beta_1\xi_L} + 1.828\,(1 - e^{-\beta_2\xi_L}) \quad (20a)$$

which gives the correct limiting behavior for small and large ξ_L [compare with equations (9) and (11), noting that $\phi_F \sim a_1\,\xi_L^{2/3}$ as $\xi_L \to 0$ and $\phi_F \sim 1 - b_1\,e^{-\alpha_1\xi_L}$ as $\xi_L \to \infty$] irrespective of the values of β_1 and β_2. Further, the RMS deviation between equation (20a) and the 'exact' result [based on equations (9) and (11)] over the range of the reported data, namely $6 \times 10^{-4} \leqslant \xi_L \leqslant 0.6$, can be made as small as 2.7% by choosing (β_1, β_2) to lie within a rather broad domain well represented by

$$\beta_1 = 4.10, \quad \beta_2 = 6.75. \quad (20b)$$

For comparison, it is noted that equation (19) fits equations (9) and (11) with an RMS deviation of 7.4% over the above domain in ξ_L whereas equation (7b) has a corresponding RMS deviation of 33.3% with respect to the exact result [the behavior of (7b) being especially poor for small ξ_L].

In examining the results in Table 2, it is noted that the last two rows indicate, respectively, the RMS deviation relative to the various correlations for all the cited data and for all the data except that of Kern and Othmer [12]. Rather surprisingly, it is seen that, amongst correlations (1)–(5), the first and earliest, due to Eubank and Proctor [15], does the best. Further, it is seen that use of the F_1 correction factor, equation (18), improves correlation (1) but worsens (2)–(5). On the other hand, the correlation based on (7) is seen to do extremely poorly whereas, by replacing equation (7b) with equation (19) for the forced-convection term,

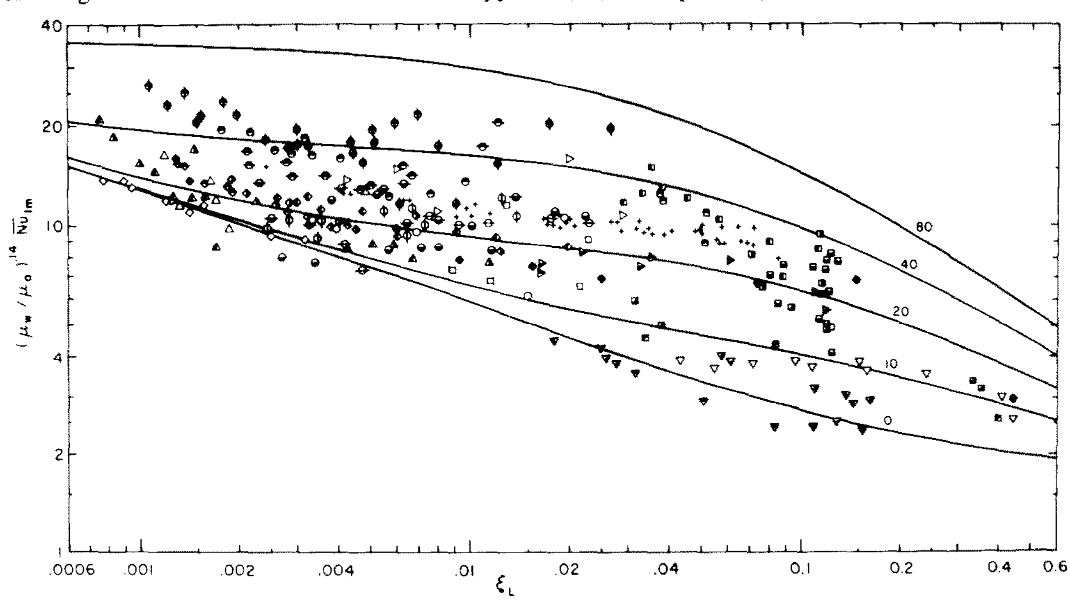

FIG. 2. Experimental results (symbols defined in Tables 1 and 2) for $(\mu_w/\mu_a)^{0.14}\,Nu_{lm}$ vs ξ_L together with curves for $(Gr\,Pr)^{1/4} = 0, 10, 20, 40$ and 80 based on equation (10) with $(Nu_F)_{lm}$ based on equation (20) and $(Nu_B)_{lm}$ on equation (17). All property values (other than μ_w) evaluated at T_a.

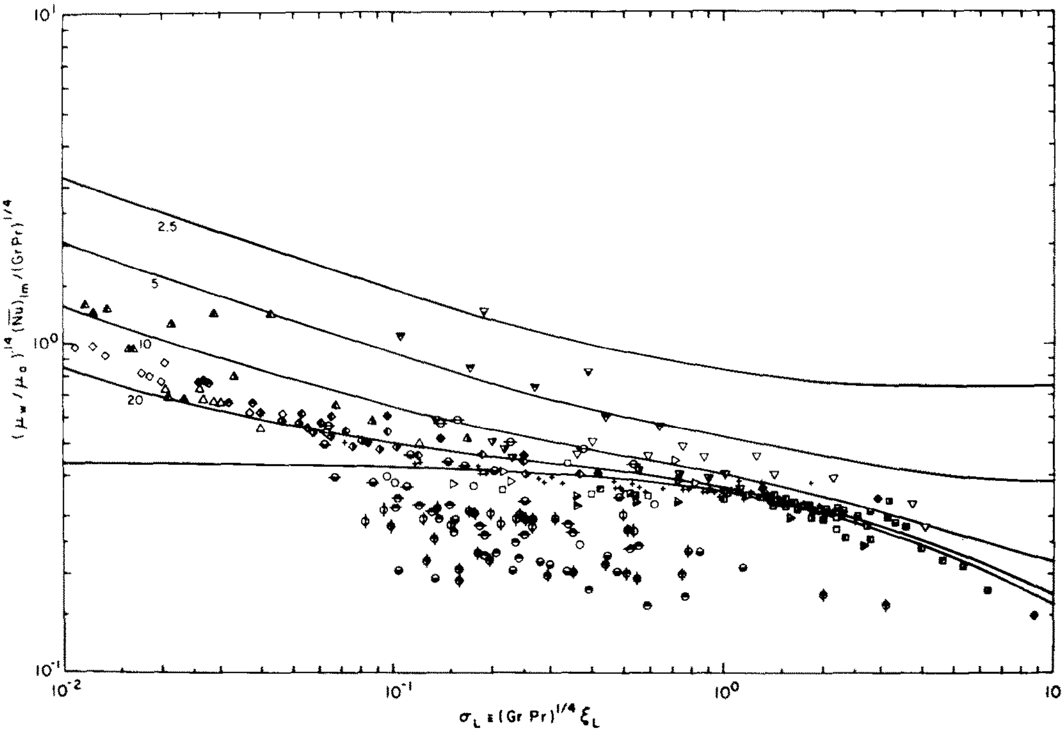

FIG. 3. Experimental results (symbols defined in Tables 1 and 2) for $(\mu_w/\mu_a)^{0.14}\,(\overline{Nu})_{lm}/(Gr\;Pr)^{1/4}$ vs σ_L together with curves for $(Gr\;Pr)^{1/4} = \infty$, 20, 10, 5 and 2.5 based upon equation (10) with $(\overline{Nu}_F)_{lm}$ based on equation (20) and $(\overline{Nu}_B)_{lm}$ on equation (17). All property values (other than μ_w) evaluated at T_a.

the agreement with the data becomes quite good. (The fact that equation (7) does so poorly suggests that the result from [19] may contain a misprint.) Lastly, it is noted that the last three correlations describe the data quite well when the results of Kern and Othmer [12] are omitted, with the best correlation corresponding to the last column of Table 2, which is represented most simply by equation (10) with $(\overline{Nu}_F)_{lm}$ based on equation (20) and $(\overline{Nu}_B)_{lm}$ on equation (17). When the data from ref. [12] are included, however, the latter correlations do not fare nearly as well, with the poorest agreement being associated with the transformer oil and cylinder oil data in the two larger diameter tubes which, from Table 1, are seen to correspond to the higher values of $(Gr\;Pr)^{1/4}$.

A more detailed comparison of the data with the correlation based upon equations (10), (17) and (20) is shown in Figs. 2 and 3. In the former, nominally every third data point is plotted in terms of $(\mu_w/\mu_a)^{0.14}\,(\overline{Nu})_{lm}$ vs ξ_L with the indicated curves based upon equations (10), (17) and (20) for $(Gr\;Pr)^{1/4} = 0$, 10, 20, 40 and 80. Figure 3 presents analogous results but with the abscissa now multiplied by $(Gr\;Pr)^{1/4}$ and the ordinate divided by the same, the curves now corresponding to $(Gr\;Pr)^{1/4} = \infty$, 20, 10, 5 and 2.5. That is, the lowest curves in Figs. 2 and 3 correspond, respectively, to the forced-convection and buoyancy-dominated asymptotes such that both should be lower bounds for the data.

In the case of Fig. 2, it is seen that the $(Gr\;Pr)^{1/4} = 0$ curve is a pretty good lower bound for the data. It is noted in particular that the data points (◇) of White correlate extremely well with the forced-convection asymptote. On the other hand, the largest systematic departure below the lowest curve in Fig. 2 is seen to correspond to the heated-glycerol data points (▽) of Oliver, which tend to lie $\approx 15\%$ below the curve. Moreover, the cooled-glycerol data points (▽), including those not shown plotted, tend to lie $\approx 10\%$ above the curve. In fact, if one omits the $(\mu_w/\mu_a)^{0.14}$ factor in processing the glycerol data, then the heated and cooled results both collapse very closely onto the forced-convection asymptote, as has been shown by Oliver himself (see Fig. 2 of ref. [1]). In other words, the glycerol data of Oliver suggest that the Sieder–Tate empiricism overpredicts the variable-viscosity effect in this case.

In Fig. 3, it is seen that the lowest curve forms a reasonable lower bound for the data excluding that of Kern and Othmer [12], the bulk of the latter lying 25–50% below the buoyancy-dominated curve. That is, whereas the plot of the data from [12] in Fig. 2 indicates heat transfer rates which are as much as four times as large as for pure forced convection, the plot of the same points in Fig. 3 indicates that the heat transfer rates are not nearly as large, for the given $(Gr\;Pr)^{1/4}$ range, as the present semi-analytical correlation would indicate. In this regard, it is noted that Kern and

Othmer observed (p. 526 of ref. [12]) that their measured values for the outlet bulk temperature were highly susceptible to external vibrations, which tends to suggest the possibility of some kind of instability. This supposition is further strengthened by noting that the poorest agreement between the data in ref. [12] and the present correlation corresponds to the larger values of $(Gr\ Pr)^{1/4}$, with the Rayleigh number (based on radius) being as large as 2×10^8. Of course, the situation is further complicated in the present case by large viscosity variations, with the inlet viscosity being as much as 100 times larger than the value at the wall. In any event, since the present semi-analytical correlation describes the data of all the remaining investigations quite well, there seems reason to suspect that the larger Rayleigh number results from ref. [12] correspond to a buoyancy-dominated structure which is different from that upon which the present correlation is based. Evidently, more work in this area would be appropriate.

As indicated in ref. [4], the data obtained in air by Jackson *et al.* [13] (detailed results presented in ref. [14]) have also been plotted (+) in Figs. 2 and 3. According to a standard procedure for gases [23], the property values have all been evaluated at the mean bulk temperature, as for the liquids, but with no viscosity-ratio factor multiplying the Nusselt number. It is to be noted that all the data from [13, 14] correspond to $(Gr\ Pr)^{1/4}$ lying between 26 and 30, such that the agreement with the present correlation is very good, as can be seen most clearly by comparing with the curves corresponding to $(Gr\ Pr)^{1/4} = 20$ and 40 in Fig. 2. That is, even though the present expression for $(\overline{Nu}_B)_{lm}$ is related to a large Prandtl number theory, the comparison with the data from refs. [13, 14] indicates that the present correlation can also describe buoyancy-dominated heat transfer in fluids for which $Pr = O(1)$.

Before ending this section, it should be noted that three sets of data have been omitted in the above comparisons, namely the shortest-tube data of Sherwood *et al.* [10], the heated oil-A data of Sieder and Tate [11] and the glycerol–water data of Depew and August [3]. Of these, the last two have been dismissed on the basis of internal inconsistencies, with the data essentially lying in two largely divergent groups in both cases. For example, for the glycerol–water data, it can be seen from Table 3 of ref. [3] that the inlet temperature and the wall temperature are essentially unchanged in all eleven runs, such that the mass flow rate ($\dot{m}$) is the only independent parameter which is varied. However, comparison of runs 1 and 7 in Table 3, corresponding to approximately the same $\dot{m}$, indicates a $\overline{Nu}_{am}$ in run 7 which is $\approx 40\%$ above that in run 1, with a similar discrepancy existing between runs 2 and 10 at a somewhat larger $\dot{m}$. In the case of the heated oil-A data, corresponding to runs 43–52 in Table I of ref. [11], it can be seen that both the inlet temperature and mass flow rate are essentially unchanged but the wall temperature is varied. However, even though runs 45 and 46 correspond to essentially the same conditions, the measured bulk-temperature rise is three times larger in run 45, with similar comparisons between runs 47 and 48 and between runs 51 and 52 indicating discrepancies by a factor of two in measured bulk-temperature rise. Given such large discrepancies, it is difficult to imagine why the original investigators did not question the reliability of such data sets. On the other hand, Sherwood *et al.* [10] did note the anomalous behaviour of the data obtained in the shortest of their four tubes, attributing this to the effect of an abrupt contraction at the tube inlet.

At this point it might also be noted that amongst all the experimental investigators reported above, only the results of Holden and of White [8] included a comparison between the heat transfer rates based upon the bulk-temperature change of the primary fluid and that associated with the secondary fluid (condensing steam, in those two cases), with runs being discarded in which the two rates did not agree within 10%. (Accordingly, in processing the data of Holden and of White, the bulk-temperature rise has been based upon an averaging of the two heat transfer rates.) Indeed, it seems unfortunate that such a procedure had not been followed in the other investigations. In fact, Kern and Othmer noted (p. 525 of ref. [12]) that it was regrettable that such a check was not feasible in their investigation "since the water flow at the minimum steady state permitted a temperature rise of only one or two degrees".

Finally, it should be noted that if the correlation for $\overline{Nu}_{lm}$, based upon (10), (17) and (20), is transformed into $\overline{Nu}_{am}$, then the corresponding comparison with the above data (omitting that from ref. [12]) reduces the RMS deviation from 11.7% to 11.0%; further, if the comparison is expressed in terms of $\overline{Nu}$ or, equivalently, ϕ, then the RMS deviation is reduced further to 9.8%. It might also be noted that changing the exponent in equation (10) from 3 to 2 or 4 results in increasing the RMS deviation from 11.7% to 14.0% or 12.7%, respectively, indicating that the cubic summation is best in this case, as advocated also by Churchill [7].

5. CONCLUSIONS

A semi-analytical correlation based upon equation (10), with $(\overline{Nu}_B)_{lm}$ given by equation (17) and $(\overline{Nu}_F)_{lm}$ by equation (20), has been developed which describes the data in refs. [1–3] and [8–11] with an RMS deviation of 11.7%. Whereas the data in these investigations correspond to $(Gr\ Pr)^{1/4} \lesssim 44$, much of the data in ref. [12] corresponds to larger values of $(Gr\ Pr)^{1/4}$, for which the present correlation does rather poorly. Hence, pending further experimental results in the large $(Gr\ Pr)^{1/4}$ domain, it seems that the present correlation should be restricted to $(Gr\ Pr)^{1/4} \lesssim 44$, where Gr is based on tube radius. Beyond this range, the best laminar correlation seems to be that of Eubank and Proctor [15], as given by equation (1), which describes the data in [1–3] and [8–12] with an RMS deviation of 21% and can be further reduced to 19% by multiplying the RHS of equation (1) by F_1, as given by equation (18).

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APPENDIX 1.

CLOSED-FORM RESULT FOR BUOYANCY-DOMINATED ASYMPTOTE

A perturbation analysis was developed in section 2(d) of ref. [5] for the interaction between the buoyancy-dominated thermal boundary layer and an assumed non-stratified core region. This resulted in a 6-term series expansion for the bulk-temperature rise as given in equation (8) for $\sigma_L \leqslant 0.70$. However, if one converts this result to the log-mean Nusselt number, making use of equation (11), it is found that $\overline{Nu}_{lm}$ is essentially constant over the range of $0 \leqslant \sigma_L \leqslant 0.70$. This can be seen from Fig. 1 in which ϕ increases from zero to 43% between $\sigma_L = 0$ and 0.70 whereas, over the same range, Nu_{lm} only changes from 0.435 to 0.405. In turn, this suggests that a simpler expansion will result if the various characteristic quantities are now scaled in terms of $\Delta T^* \equiv T_w - T_b(z)$ rather than $\Delta T \equiv T_w - T_0$.

In fact, if one returns to the "intermediate region" analysis in section 2(b) of ref. [5] and replaces T_0 everywhere with $T_b(z)$ such that the Grashof number appearing in the definitions of δ_B and V_B is now based on $|\Delta T^*|$ and the temperature distribution [relative to $T_b(z)$] is now normalized with respect to ΔT^*, the resulting problem for "$f(\eta)$" and "$h(\eta)$" remains unchanged with the circumferentially averaged heat flux given by equation (2.25) of ref. [5] but where ΔT and "G" (Gr, in present notation) are now in terms of ΔT^*. Hence, since $\Delta T^* = \Delta T(1-\phi)$, it follows that the net effect is the introduction of an additional $(1-\phi)^{5/4}$ factor on the RHS of equation (2.25). Accordingly, the resulting local energy-rate balance becomes

$$2\pi a\left[0.43526\frac{k\,\Delta T}{a}(Gr\,Pr)^{1/4}(1-\phi)^{5/4}\right] = \dot{m}C_p\,\Delta T\frac{\mathrm{d}\phi}{\mathrm{d}z} \tag{A1}$$

which reduces to

$$(1-\phi)^{-5/4}\frac{\mathrm{d}\phi}{\mathrm{d}\sigma} = C_1 \tag{A2}$$

and integrates to

$$\phi = 1-\left(1+\frac{C_1}{4}\sigma_L\right)^{-4} \tag{A3}$$

In particular, a Taylor series expansion of equation (A3) about $\sigma_L = 0$ results in

$$\phi(\sigma_L) = C_1\sigma_L - 10\left(\frac{C_1\sigma_L}{4}\right)^2 + 20\left(\frac{C_1\sigma_L}{4}\right)^3 - 35\left(\frac{C_1\sigma_L}{4}\right)^4$$

$$+56\left(\frac{C_1\sigma_L}{4}\right)^5 - 84\left(\frac{C_1\sigma_L}{4}\right)^6 +$$

$$= 0.87052\sigma_L - 0.47363\sigma_L^2 + 0.20615\sigma_L^3$$

$$- 0.07851\sigma_L^4 + 0.02734\sigma_L^5 - 0.00892\sigma_L^6 + \ldots$$

which agrees with the C_n from equation (2.49) of ref. [5], as given in equation (8b) of present text, thus obviating the need for section 2(d) of ref. [5].

APPENDIX 2.

SOURCES OF PROPERTY VALUES

Reducing the data reported in refs. [1–3] and [8–12] requires values for the density, viscosity, thermal conductivity, specific heat and volumetric coefficient of thermal expansion as functions of temperature. For the water runs in refs. [1–3] there is apparently no problem in this regard. Further, for the three oils employed in ref. [12], the authors plot straight-line curves for log μ (10^{-2} g cm^{-1} s^{-1}) vs T(°F) and also for $C_p(T)$, $k(T)$ and $\rho(T)$ [actually, specific gravity, which $\simeq \rho(T)/0.999$]. However, cross-checking these plots with results in Kern [24] [in particular, the specific gravity at 60°F can be used to determine the °API, based upon equation (1.4) on p. 4 of ref. [24], from which one can then determine $k(T)$ and $C_p(T)$ from Fig. 1 on p. 803 and Fig. 4 on p. 806, respectively] indicates that the $C_p(T)$ curves in Fig. 3 of ref. [12] have been mislabelled such that "transformer oil dist." and "core oil" should be switched. Further, there seems strong reason for suspecting that the $\mu(T)$ curves in Fig. 3 of ref. [12] have also been mislabelled, since $\mu(T)$ typically decreases with increasing °API (as in the case of the three oils considered by Sieder and Tate [11]), such that "cylinder oil" and "core oil" should be switched in the $\mu(T)$ plots in ref. [12]. For the three oils in [11], for which $\mu(T)$ and the °API are explicitly given, one can generate $k(T)$, $C_p(T)$ and $\rho(T)$ [hence, $\beta(T)$] from Figs. 1, 4 and 6 on pages 803, 806 and 809, respectively, of ref. [24]. Further, for the light oil employed in ref. [10], for which $\mu(T)$ is tabulated and ρ at 15°C $\simeq$ 60°F is given, one can use the latter to calculate °API and, hence, $k(T)$, $C_p(T)$ and $\rho(T)$ from ref. [24]. A similar procedure can also be employed for the light oil (Velocite B) used by Holden and by White, as reported in ref. [8].

On the other hand, property values for the glycerol, ethyl alcohol and glycerol–water solution used by Oliver [1] together with the ethyl alcohol–water solution of Depew and August [3] and the glycerol–water solution of Rynalski and Huntington [9] require more attention. In particular, concerning the most temperature sensitive of the properties, namely the viscosity, the values have been based upon Segur and Oberstar [25] for glycerol, upon the CRC Handbook [26] for ethyl alcohol, on the Ethyl Alcohol handbook [27] for the ethyl alcohol–water solution of Depew and August [3], on Segur and Oberstar [25] for the glycerol–water solution of Rynalski and Huntington [9] and on Oliver [1] for his own glycerol–water solution. It is noted that Oliver indicates his solution to have been approximately 80% by weight glycerol in water; in fact, by comparing his $\mu(T)$ values with those of Newman [28] and his density value at 20°C with those in ref. [26], it follows that a more exact value would be 77% glycerol by weight. Further, Depew and August indicate that their ethyl alcohol was 95% pure by volume, thus suggesting $\approx$94% by weight. Other sources of property values have included Touloukian [29] for thermal properties of glycerol and ethyl alcohol, Miner and Dalton [30] for thermal conductivity of glycerol–water solutions, Bosart and Snoddy [31] for density of glycerol and its solution together with some density and coefficients of thermal expansion from Vargaftik [32] and the Critical Tables [33].

CONVECTION MIXTE LAMINAIRE DAHS UN TUBE HORIZONTAL ET ISOTHERME: FORMULATION DES DONNEES DE TRANSFERT THERMIQUE

Résumé—Une formule semi-analytique pour la convection mixte dans un tube isotherme et horizontal est développée pour décrire toutes les données disponibles de transfert thermique (à l'exclusion de celles de Kern et Othmer), avec un écart-type de 11,7% basé sur une moyenne logarithmique du nombre de Nusselt, de 11% basé sur la moyenne arithmétique du nombre de Nusselt, et de 9,8% basé sur l'élévation de température fractionnaire. Le fait que la formule ne décrive pas les données de Kern et Othmer est attribué aux grands nombres de Rayleigh, ce qui nécessite une recherche ultérieure. Bien qu'établie pour des fluides à grand nombre de Prandtl, la formule décrit aussi bien le transfert dans l'air, dominé par les forces d'Archimède.

LAMINARE MISCH-KONVEKTION IN EINEM ISOTHERMEN HORIZONTALEN ROHR: KORRELATION VON WÄRMEÜBERGANGS-DATEN

Zusammenfassung—Für die laminare Misch-Konvektion in einem isothermen horizontalen Rohr wird eine halb-analytische Korrelation entwickelt. Diese ist in der Lage, alle verfügbaren Wärmeübergangs-Daten (außer denen von Kern und Othmer) mit einer mittleren quadratischen Abweichung zu beschreiben, die für die logarithmierte mittlere *Nu*-Zahl 11,7%, für die arithmetische mittlere *Nu*-Zahl 11,0% und für den relativen Anstieg der Mitteltemperatur 9,8% beträgt. Die Tatsache, daß die Korrelation die meisten der Kern/Othermer-Daten nicht beschreiben kann, ist auf die zugehörigen hohen Rayleigh-Zahlen zurückzuführen, für die weitere Untersuchungen erforderlich sind. Obwohl die Korrelation für Fluide mit großen Prandtl-Zahlen abgeleitet wurde, vermag sie ebenso gut den durch Auftrieb bestimmten Wärmeübergang in Luft zu beschreiben.

ЛАМИНАРНАЯ СМЕШАННАЯ КОНВЕКЦИЯ В ИЗОТЕРМИЧЕСКОЙ ГОРИЗОНТАЛЬНОЙ ТРУБЕ. ОБОБЩЕНИЕ ДАННЫХ ПО ТЕПЛОПЕРЕНОСУ

Аннотация—Получена полуэмпирическая обобщающая зависимость для ламинарной смешанной конвекции в изотермической горизонтальной трубе и найдено, что с ее помощью можно описывать все имеющиеся данные по теплообмену (за исключением данных Керна и Отмера) со среднеквадратичной ошибкой в 11,7%, если в основу положено среднелогарифмическое значение числа Нуссельта, 11,0%—для среднеарифмитического значения числа Нуссельта и 9,8—процентная ошибка для увеличения температуры в объеме. Указанное исключение объясняется большими значениями числа Рэлея и требует дальнейшего исследования. Хотя зависимость получена для жидкости с большим числом Прандтля, показано, что она хорошо описывает и данные по теплообмену воздуха при существенном влиянии сил плавучести.